Research Summary

Dr Xingzheng WU

Post Ref. No: 1853-07-E CED-SES-Southampton

Research Summary (1997-2007)

Part A CFRD (1997-2001) in Dalian

Three-dimensional finite element analysis of Concrete Faced-slab Rockfill Dam (CFRD)

Part B SADSS (2001-2004) in Beijing

Safety Assessment and Decision Support Software System of Dike Ring (SADSS)

Part C CLIEACH (2004-2005) in Dundee

A Process-Based Numerical Model to Predict Coastal Cliff-Beach Erosion (CLIEACH)

Part D SDEM (2005-2008) in Dundee

Stochastic Differential Equation Models for the shoreline evolution (SDEM)

Part A CFRD - loading mechanism of BSM

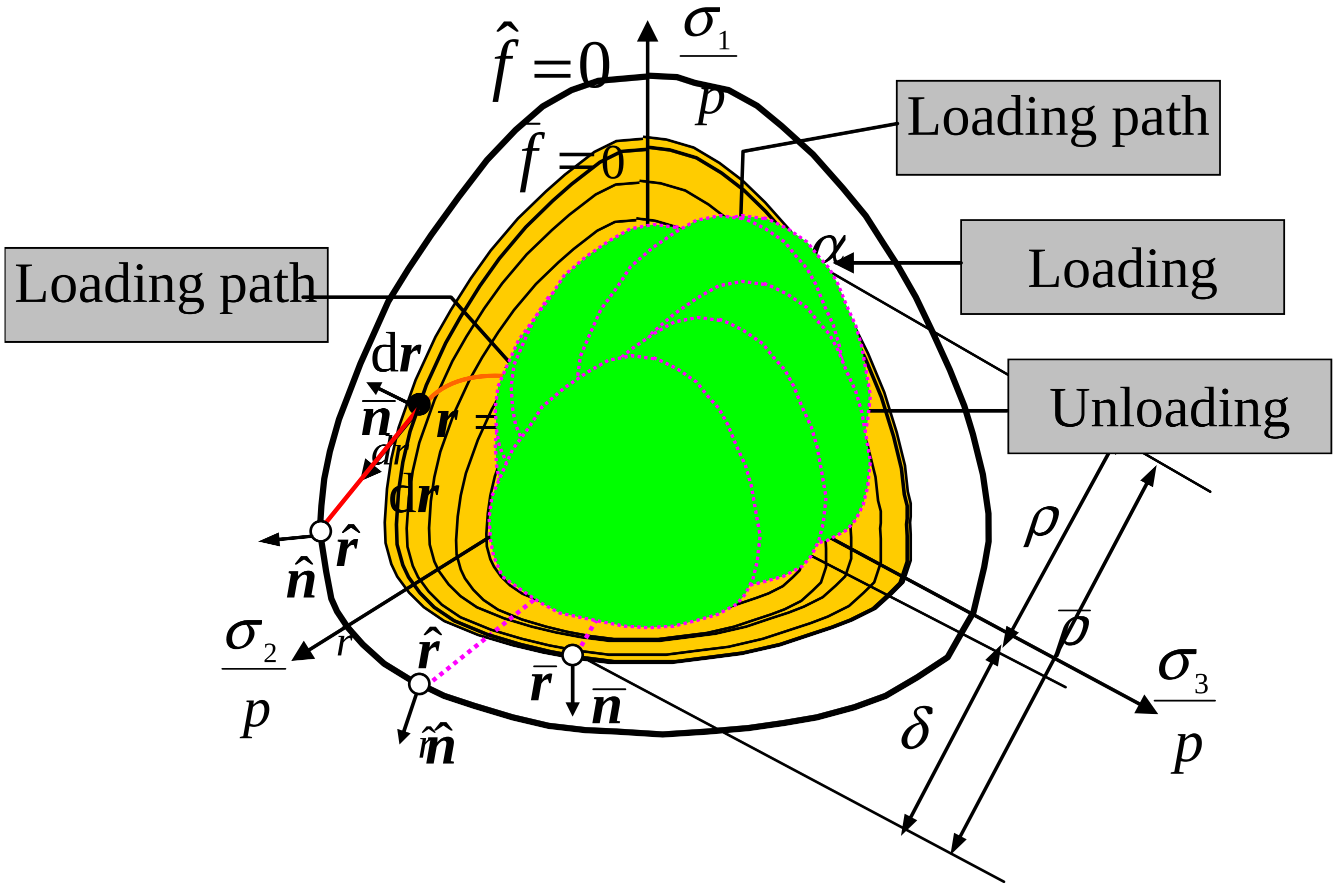
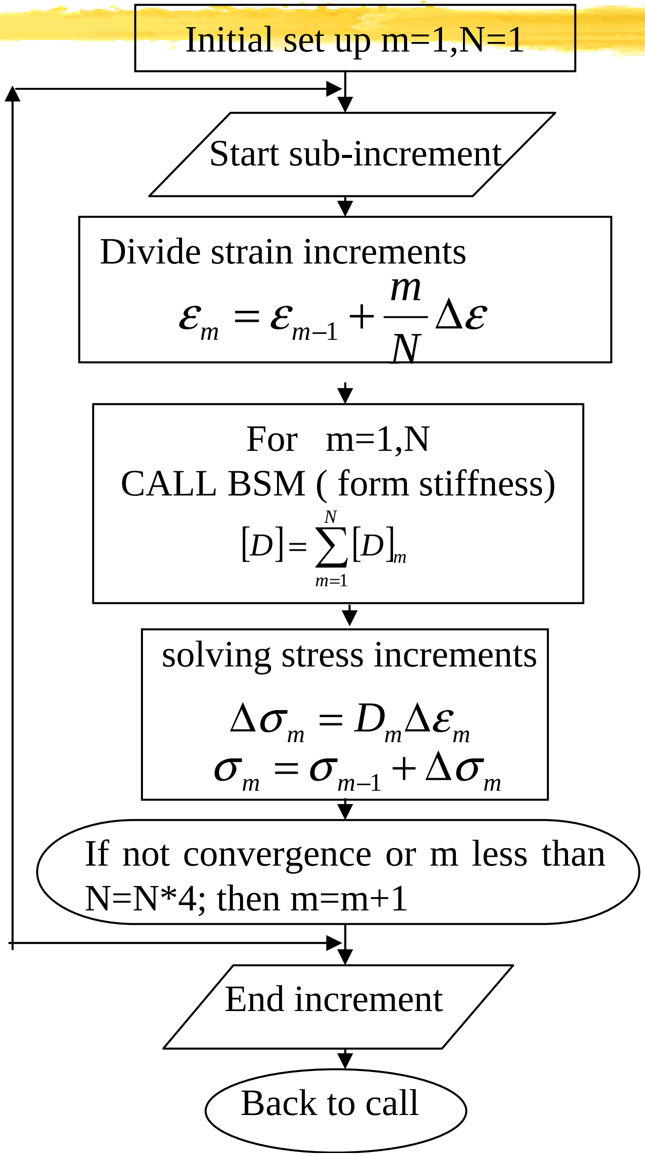


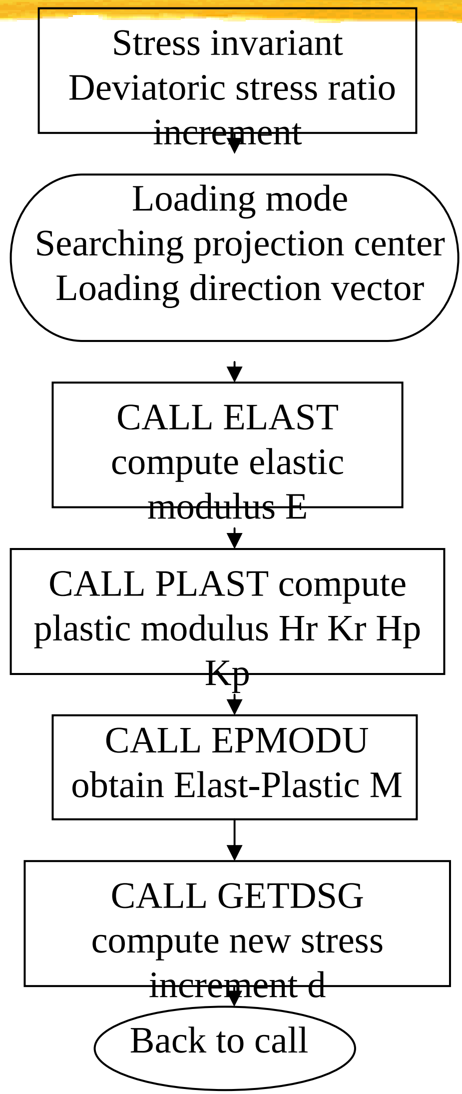
Illustration of neutral loading and unloading mechanism of the BSM in the deviatoric stress ratio space

Part A CFRD - Adaptive multi-step backward Euler's integration procedure with local iteration

BSM adaptive multistep integral



Subroutine BSM flow chart



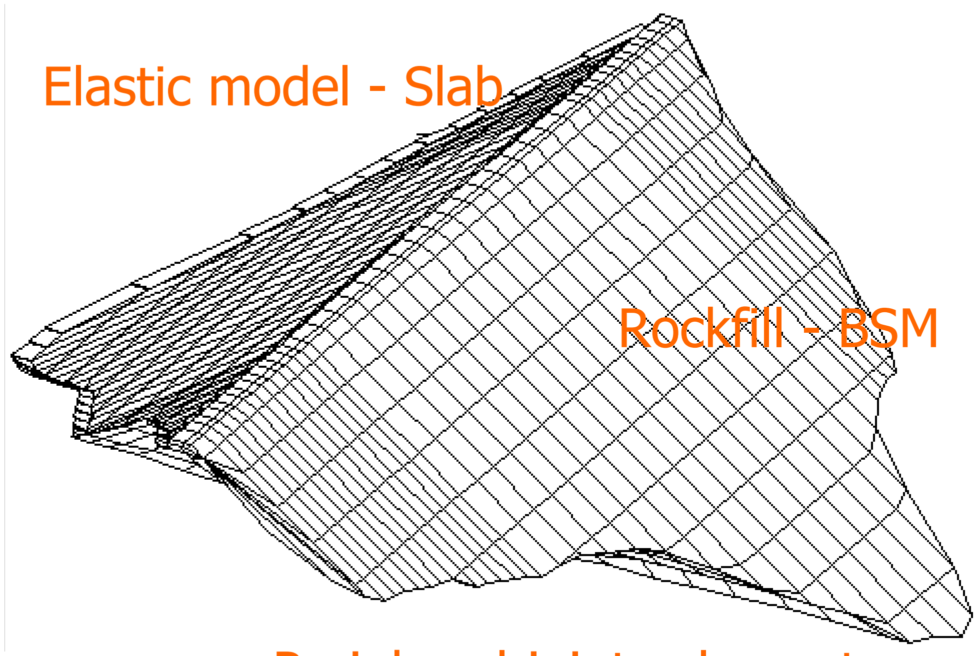
Part A CFRD - An incrementally-iterative algorithm of 3D nonlinear systems

- an incrementally-iterative algorithm to solve the nonlinear system equilibrium equations during the loading stages, including construction and water filling phase as well as seismic response to earthquake
- Wilson-θ's numerical integration scheme is combined with this procedure to solve the dynamic equations of the system step-by-step in time domain. For example the equilibrium equation of displacement increments

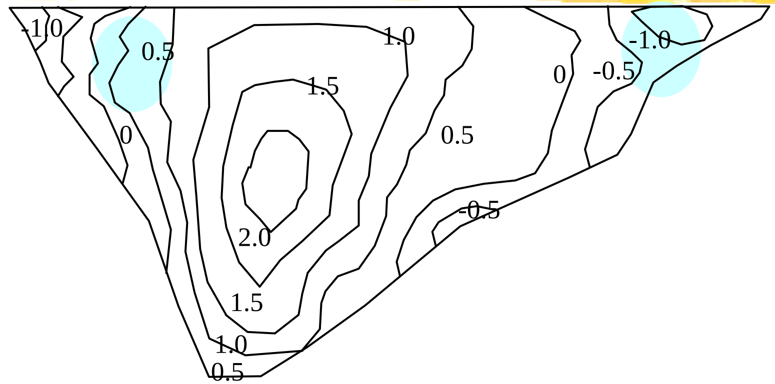
$$\left(\frac{6}{\theta^2\Delta t^2}\boldsymbol{M}+\frac{3}{\theta\Delta t}\boldsymbol{C}+{}^{t+\theta\Delta t}\boldsymbol{K}\right){}^{t+\theta\Delta t}\Delta\boldsymbol{U}={}^{t+\theta\Delta t}\Delta\boldsymbol{R}+\left(\frac{6}{\theta\Delta t}\boldsymbol{M}+3\boldsymbol{C}\right){}^{t}\dot{\boldsymbol{U}}+\left(3\boldsymbol{M}+\frac{\theta\Delta t}{2}\boldsymbol{C}\right){}^{t}\ddot{\boldsymbol{U}}-{}^{t+\theta\Delta t}\boldsymbol{F} \quad (1)$$

- Displacement increment and equivalent nodal forces at time t to $t+\theta\Delta t$ can be solved by tangent stiffness matrix approximately ${}^{t}\hat{\boldsymbol{K}}$ will meet for the geometry and material condition at t
- A modification of Newton iterative procedure (Global) was employed to capture the nonlinear characteristic of the system
- Stress and non-equivalent nodal forces of the elements will be updated by the solved displacement increments
- Final displacements can be obtained when it convergence each time step

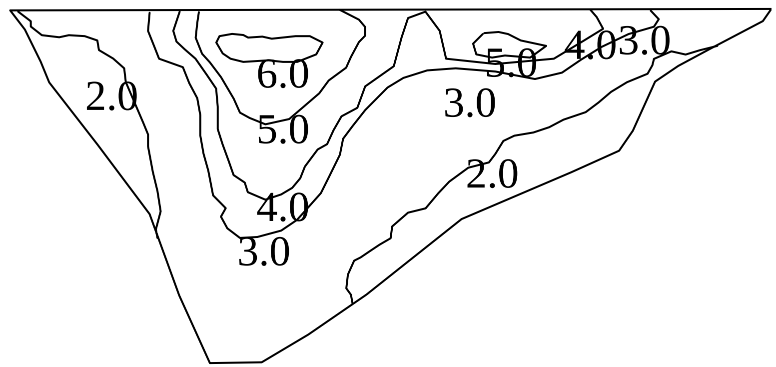
Part A CFRD - Static and dynamic response



The three-dimensional finite element discretization of a CFRD dam (182m high)

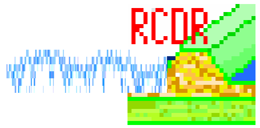
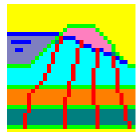
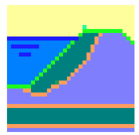


Contours of stresses on slab for reservoir filling (MPa)



Contours of peak absolute acceleration of slab (g)

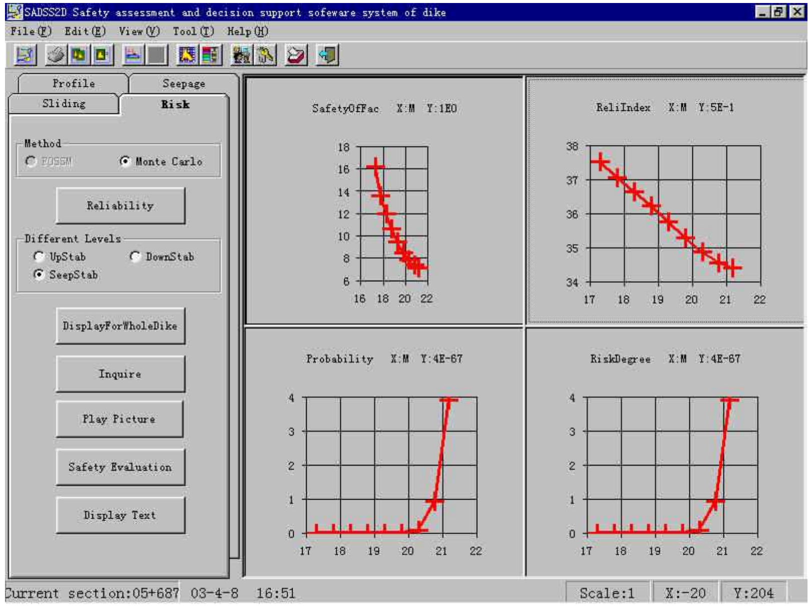
Part B SADSS Limit state equation

Failure mode	Limit state equation	Icon
Overtopping	$z_1 = h_0 - h_w - h_s - e$	
Piping	$z_2 = \gamma_{nk} d_{ks} - \gamma_w h_{ap} + \gamma_{sb} t_{sb}$	
Sliding	$z_3 = F_{SL} - 1 = M_r / M_o - 1$	

Repeat random sampling independently by MC method

$$\hat{P}_f = \frac{1}{N} \sum I(g(x_1, x_2, \cdots, x_n) \le 0) = \frac{M}{N} \qquad (2)$$

Part B SADSS---Various evaluation index



Part B SADSS Distribution of indexes

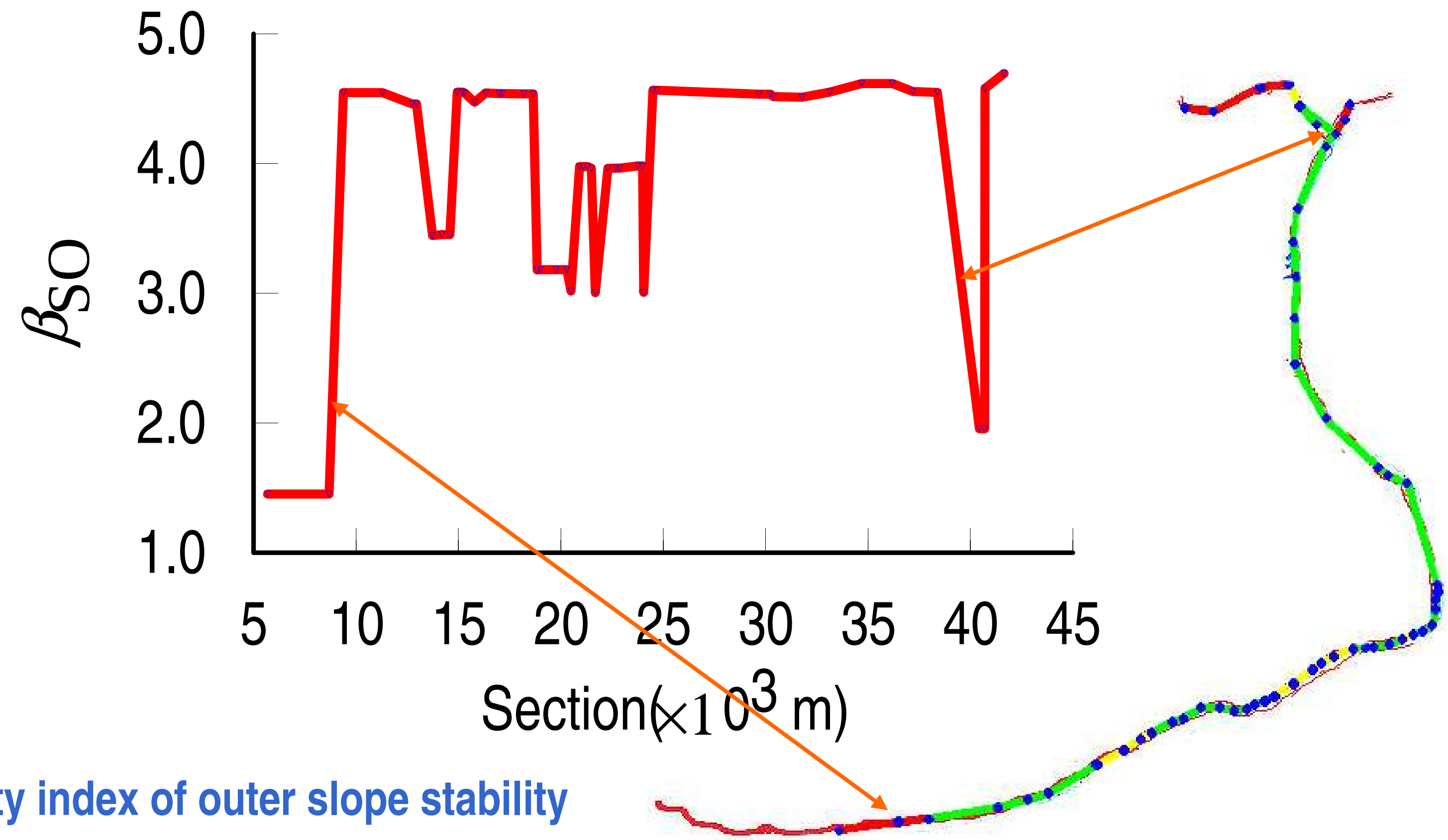
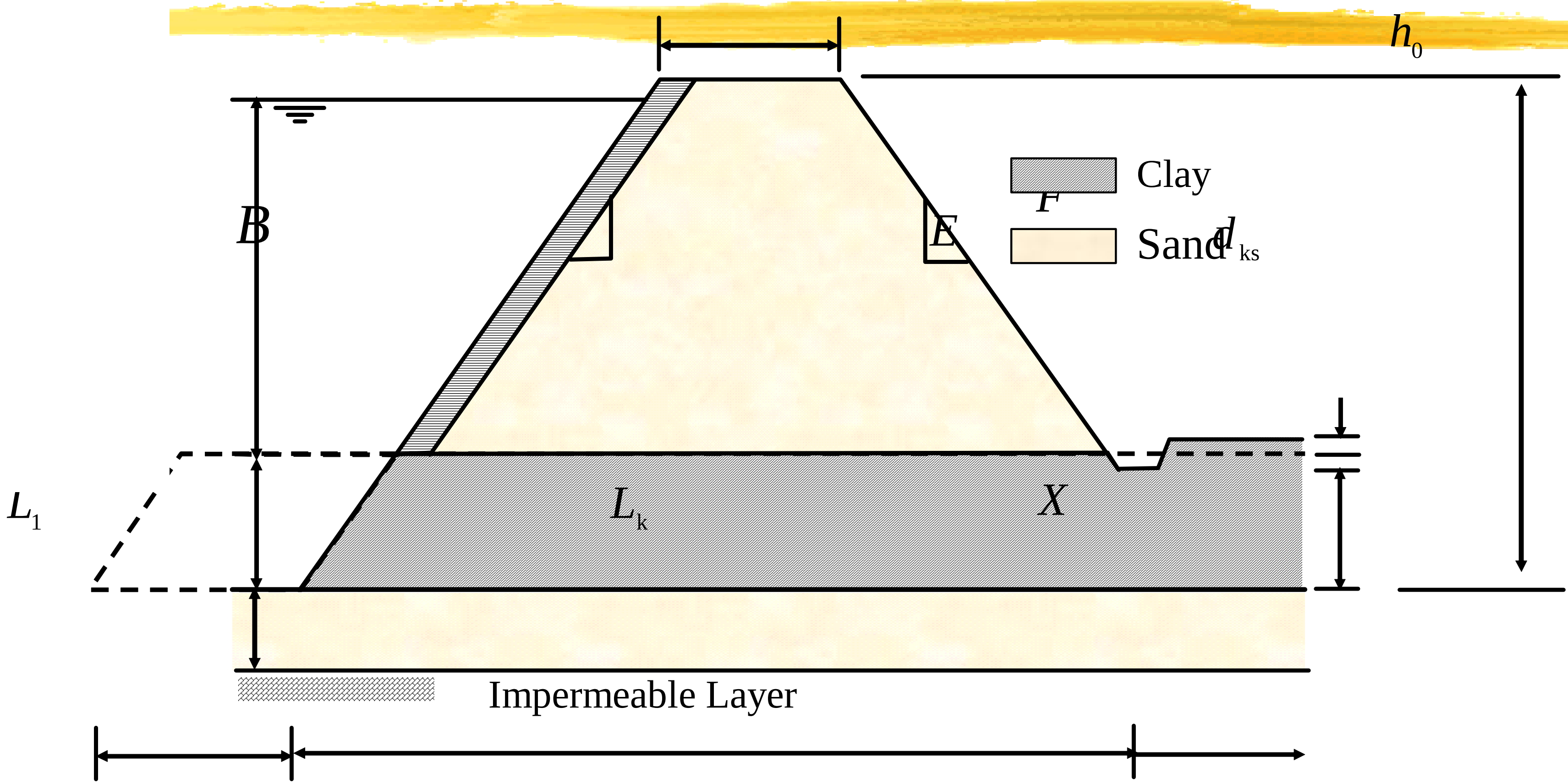


Fig. Reliability index of outer slope stability

The fundamental lower and upper bounds:

$$\max\_P\{Z_i < 0\} \leq P\{dike\_fails\} \leq \sum_{i=1}^{55} P\{Z_i < 0\} \qquad (3)$$

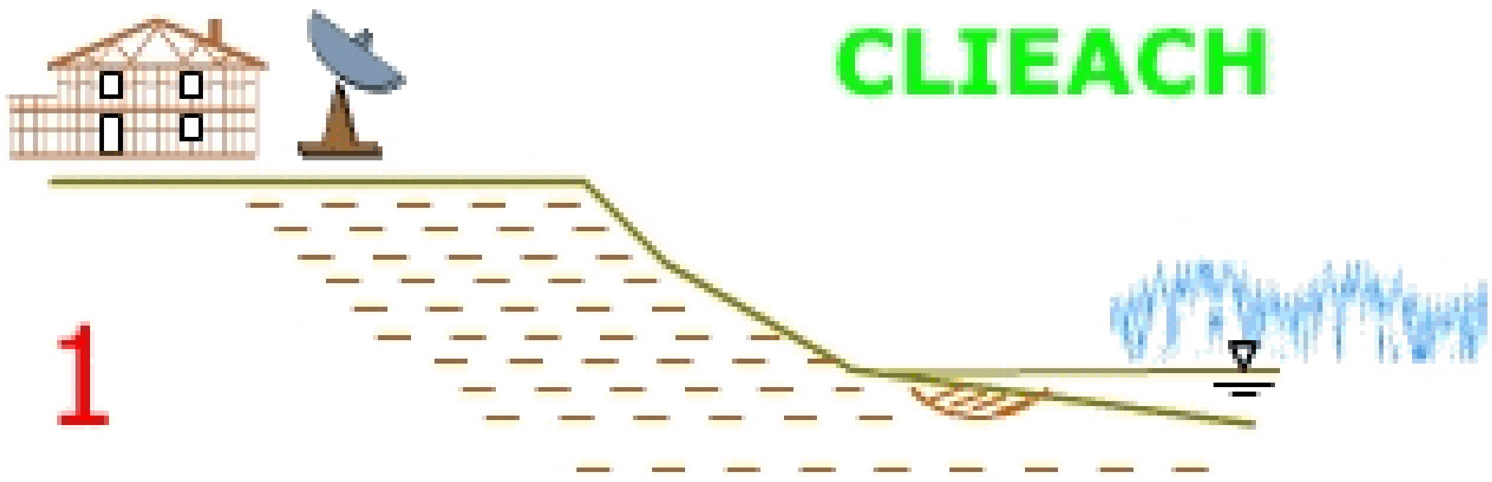
Part B SADSS --- Probabilistic optimum design



An idealized cross section with inclination facing on two-phase fluvial facies basement

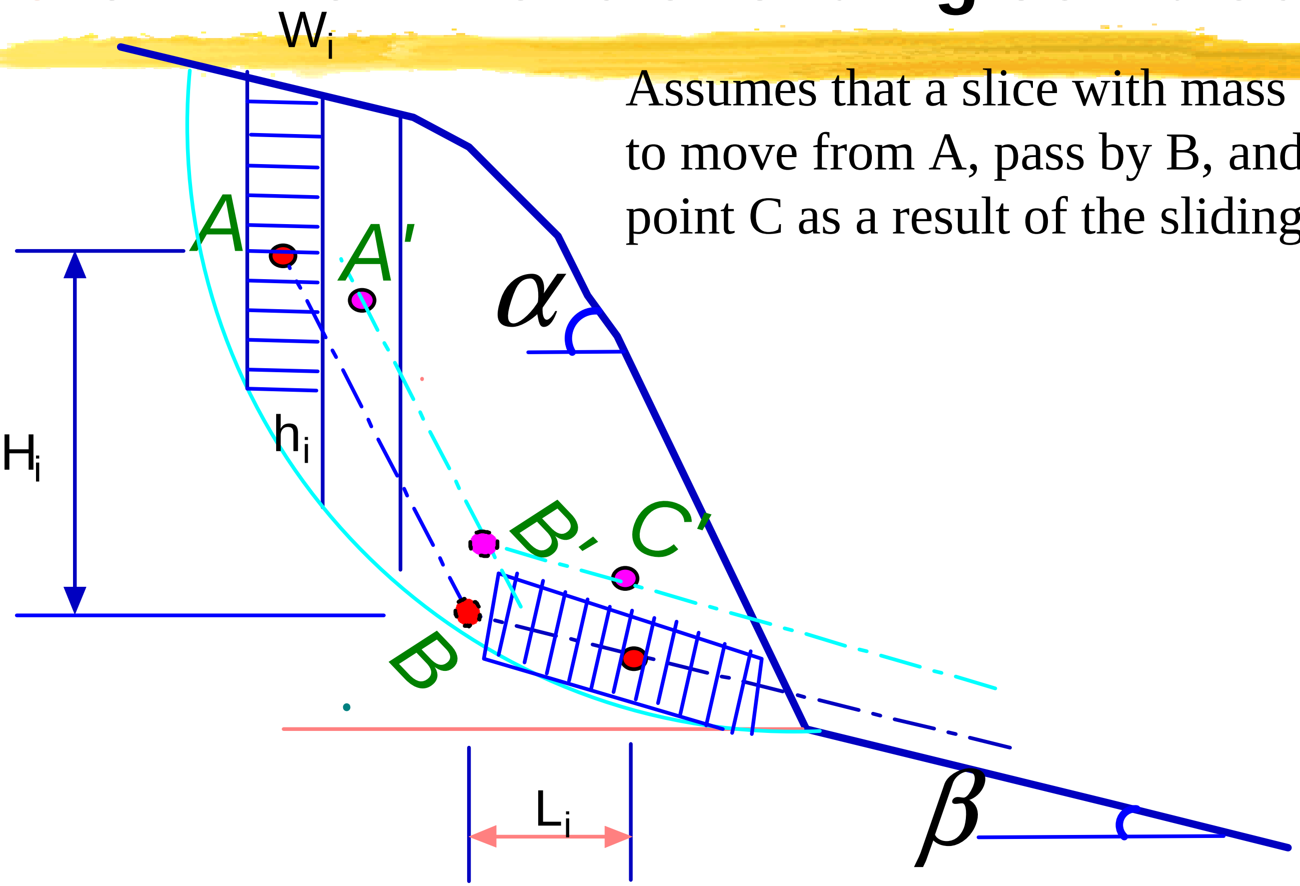
O.F. $$\min f_O(h_f) = IC_O + \sum_{t=0}^{N_y} \frac{1}{(1+i_R)^t}(L_P + RC_O) - SV \qquad (4)$$

Part C CLIEACH Stages of retrogression



Continual wave action transport the debris offshore. The cliff base is again exposed to waves to suffer erosion >>> Centrifuge test

Part C CLIEACH Move of sliding soil block



Assumes that a slice with mass M starts to move from A, pass by B, and moves to point C as a result of the sliding

neglect deformation characteristics during the moving

Two phases: AB is the transferring phase of potential energy to kinetic energy overcome friction, accelerating motion while BC is the exchange phase of kinetic energy overcome the friction.

Part C CLIEACH Governing equation

Energy conservation equation

$$\gamma w_i h_i H_i - \gamma w_i h_i \frac{H_i}{\tan\alpha}\tan\phi_\alpha = \frac{1}{2}\frac{\gamma}{g} w_i h_i v_B^2 \qquad (5)$$

$$\frac{1}{2}\frac{\gamma}{g} w_i h_i [v_B \cos(\alpha-\beta)]^2 + \gamma w_i h_i L_i \tan\beta = \gamma w_i h_i L_i \tan\phi_\beta \qquad (6)$$

Run out distance of the sliced block

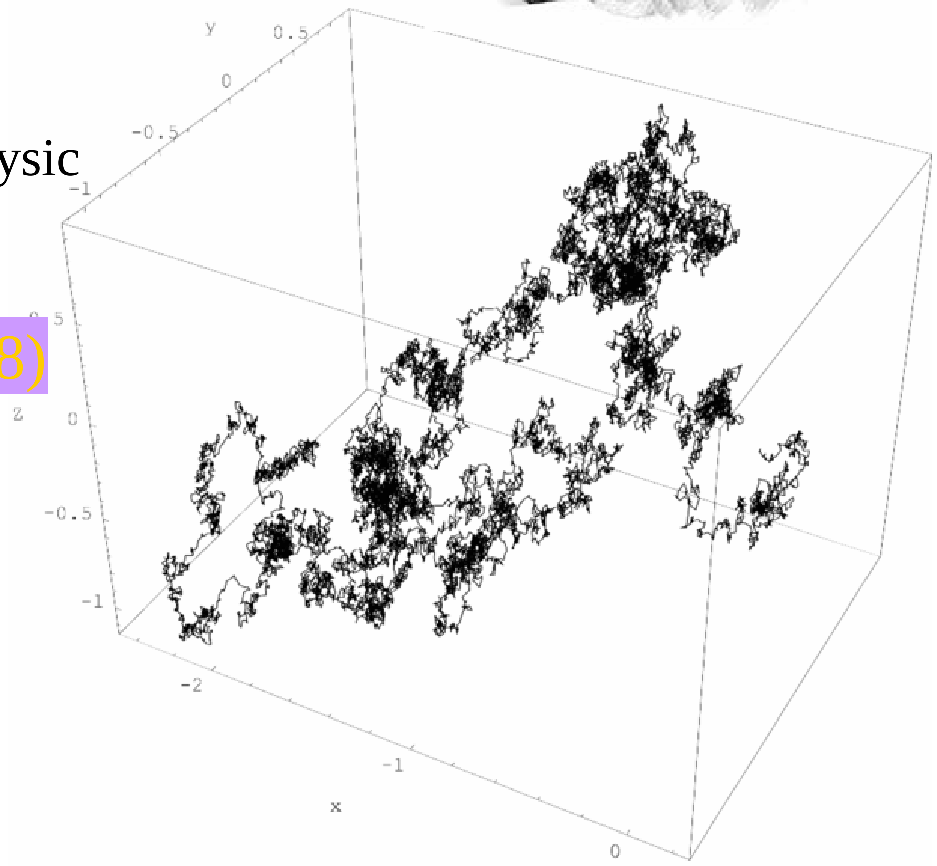
$$L_i = H_i \frac{\cos^2(\alpha-\beta)}{\tan\alpha} \frac{\tan\alpha - \tan\phi_\alpha}{\tan\phi_\beta - \tan\beta} \qquad (7)$$

The height of lying of slid mass (Distribution homogenise)

$$L = \frac{\sum_{i=1}^{n}(L_i * \gamma * W_i * h_i)}{\sum_{i=1}^{n}(\gamma * W_i * h_i)} \qquad \Delta h = V_s / L$$

Part D SDEM Brownian motion

Brownian motion (Wiener process) in three-dimensional space (one sample path shown) is an example of an Itō diffusion. (from Wikipedia)



Langevin equation is a SDE in statistic physic describing Brownian motion

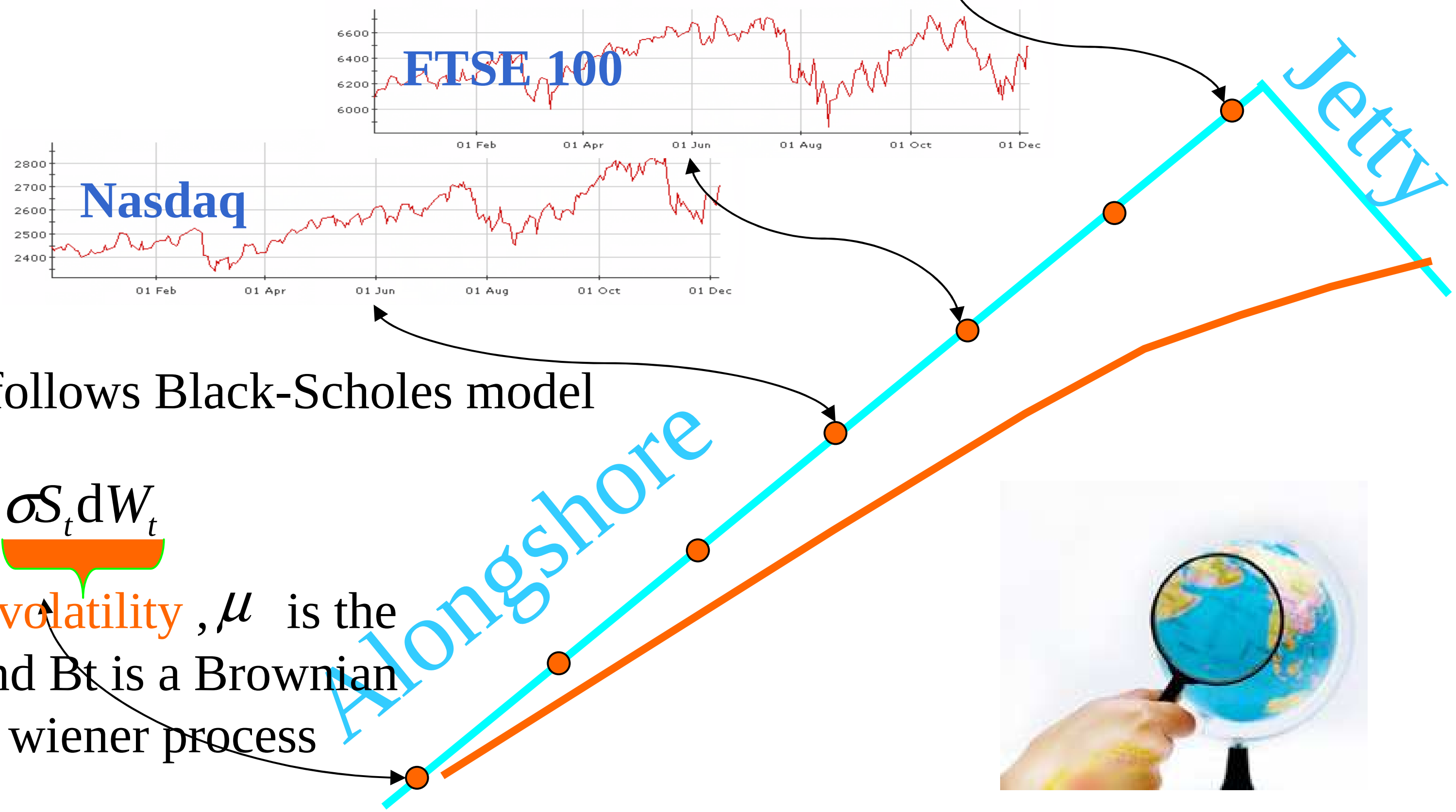
$$\frac{dy_t}{dt} = u(t, y_t) + \sigma(t, y_t)\frac{dB_t}{dt} \quad (8)$$

$$W_t = {dB_t}/{dt}$$ Wt is wiener process

Which is the random movement of particles suspended in a fluid or the mathematical model used to describe such random movements, often called a [partical theory].

Part D SDEM Idea comes from stock market

The mathematical model of Brownian motion has several real-world applications. hydraulic, environmental, and biological fields An often quoted example is stock market fluctuations



stock price S_t follows Black-Scholes model

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

σ is the stock volatility , μ is the stock **drift**, and Bt is a Brownian motion. Wt is wiener process

Part D SDEM One contour line model

Only longshore transport component considered as a contribution

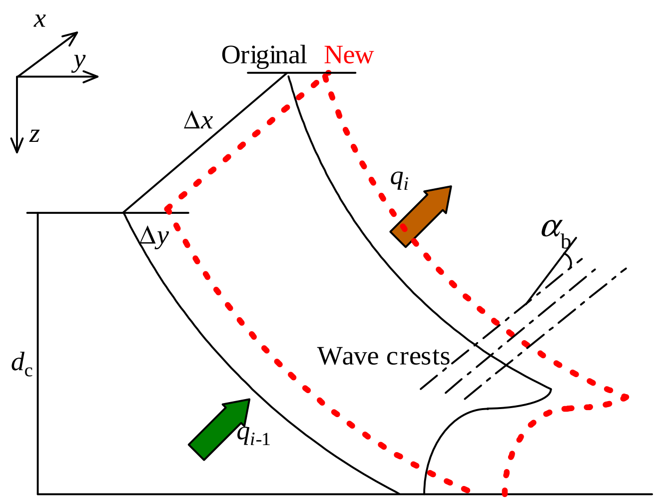
The general equation for the deterministic process

$$\frac{\partial y(x,t)}{\partial t} = -\frac{1}{D_c}\frac{\partial q_l(x,t)}{\partial x} \quad (9)$$

$$q_l(x,t) = q_{l0}(x,t)\sin\alpha_b(x,t)\cos\alpha_b(x,t)$$

$$q_{l0} = H^{5/2}\frac{1}{16}\rho_w g^{3/2}\gamma^{1/2}a_1 \quad (10)$$

$$H_w = H^{5/2} = K_m + \sigma\xi(t) = K_m + W(t)$$



Part D SDEM Stochastic one-line model

Substitute one-line model to random differential equation, then

a random process model of shoreline position can be given

$$\phi(y,t) = \frac{1}{16}\rho_{\mathrm{w}} g^{3/2}\gamma^{1/2} a_1 (\sin 2\alpha_{\mathrm{b}}{}^{i-1} - \sin 2\alpha_{\mathrm{b}}{}^{i})/(\Delta x d_{\mathrm{c}}) \qquad (11)$$

$$\frac{\mathrm{d}y(x,t)}{\mathrm{d}t} = \phi(y,t)K_{\mathrm{m}} + G(y,t)W(t) \qquad (12)$$

Drift term (mean) & random diffusion (variations)

It is known that system described by equation (12) has a transition PDF (Soong, 1973; Gardiner, 2004), satisfies the **Fokker-Planck equation**

$$\frac{\partial p(y,t)}{\partial t} = -\frac{\partial}{\partial y}\left[\phi(y,t)K_{\mathrm{m}}\, p\right] + \frac{1}{2}\frac{\partial^2}{\partial y^2}\left[(GDG^T)\, p\right] \qquad (13)$$

determining the PDF is transformed into solving above PDE

Part D SDEM PDF in 3D space at 80th cell

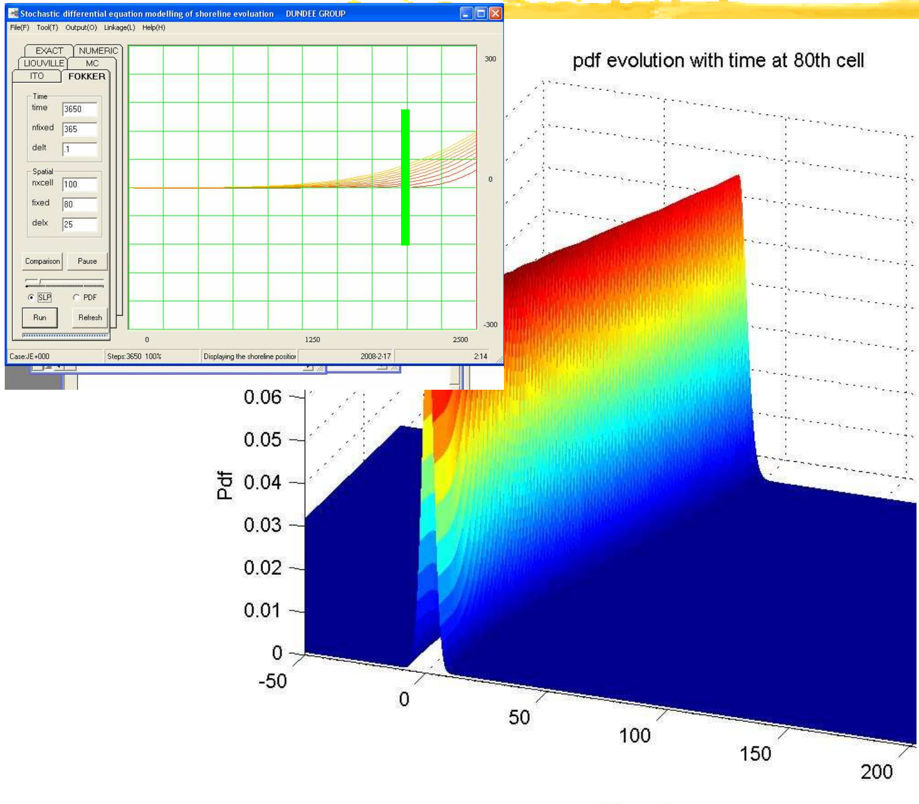


Fig. 7a PDF surface varying with time at cell 80

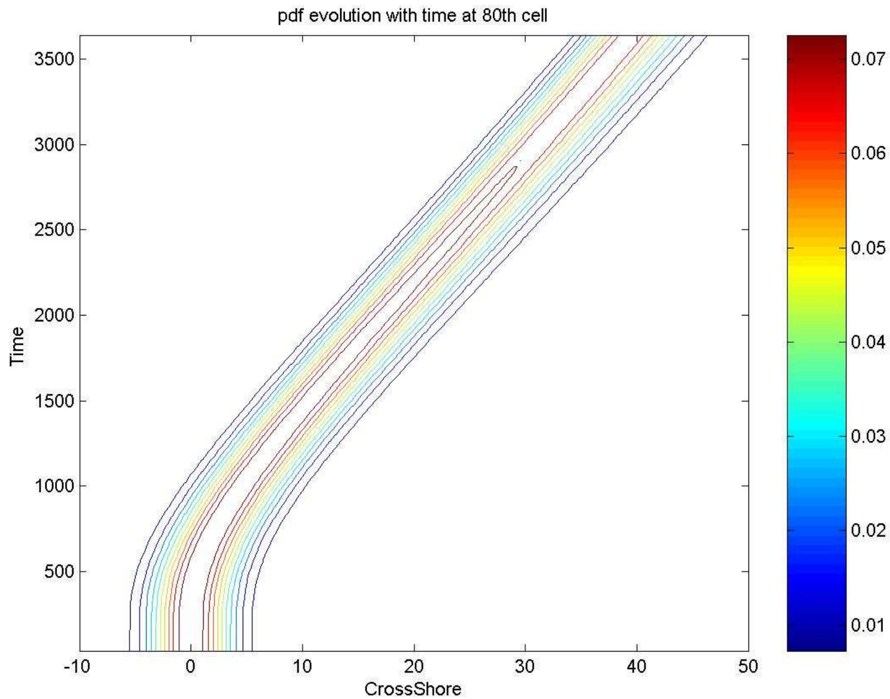


Fig. 7b The coutours of PDF at 80th

Seems like water flowing in a river with a gradient slope. The flow in the "river" is not stationary, initially no change due to no deposition at the beginning then drift constantly over time

Part D SDEM Analytical solution on Fokker-Planck

For a special transect $x = x_0$

$$y(x_0,t) = \tan\alpha_0 \sqrt{\frac{4\varepsilon H_w t}{\pi}} e^{-\frac{x_0^{\,2}}{4\varepsilon H_w t}} - x_0 \tan\alpha_0 + x_0 \tan\alpha_0 \int_0^{\frac{x_0}{2\sqrt{\varepsilon H_w t}}} e^{-\gamma^2} \mathrm{d}\gamma \quad (14)$$

we should like to explain its derivative with respect to time

$$\mathrm{d}y / \mathrm{d}t = \tan\alpha_0 \sqrt{\frac{\varepsilon H_w}{\pi t}} e^{-\frac{x_0^{\,2}}{4\varepsilon H_w t}} \quad (15)$$

This transformation of the shoreline change rate at a specified position, seems to escalate the complexity of the problem dramatically, actually this expression is expected to exhibit more interesting properties in the follow stochastic model

Fokker-Planck equation

$$\frac{\partial p(y,t)}{\partial t} = -\frac{\partial}{\partial y}\left[u(t)p\right] + \frac{\partial^2}{\partial y^2}\left[v^2(t)p\right] \quad (16)$$

At a specified position, a more promising and alternative exact solution can be solved by Lie-Algebra approach (Desai and Zwanzig, 1978; Lo, 2005) for this time-dependent

Part D SDEM Analytical solution on Fokker-Planck

We may define the evolution operator

$$p(y,t) = U(t)\,p(y,0)$$

$$\frac{\mathrm{d}U(t)}{\mathrm{d}t} = A(t)U(t), \qquad U(0)=1 \qquad A(t) = -\mu(t)\frac{\partial}{\partial y} + \nu^2(t)\frac{\partial^2}{\partial y^2} \qquad U(t) = \exp\left[\alpha(t)\frac{\partial}{\partial y}\right]\exp\left[\gamma(t)\frac{\partial^2}{\partial y^2}\right]$$

$$\alpha(t) = -\int_0^t \mu(s)\mathrm{d}s \qquad \gamma(t) = \int_0^t \nu^2(s)e^{\alpha(s)}\mathrm{d}s$$

Solution is Gaussian type with time-varying mean $\alpha(t)$ variance $\gamma(t)$

$$p(y,t) = \exp\left[\alpha(t)\frac{\partial}{\partial y}\right]\times\exp\left[\gamma(t)\frac{\partial^2}{\partial y^2}\right]p(y,0)$$

$$= \exp\left[\alpha(t)\frac{\partial}{\partial y}\right](4\pi\gamma(t))^{-1/2}\times\int_{y_{\min}}^{y_{\max}} p(z,0)\exp\left[-\frac{(z-y)^2}{4\gamma(t)}\right]\mathrm{d}z \qquad (17)$$

$$= (4\pi\gamma(t))^{-1/2}\times\int_{y_{\min}}^{y_{\max}} p(z,0)\exp\left[-\frac{(z-(y+\alpha(t)))^2}{4\gamma(t)}\right]\mathrm{d}z$$

Part D SDEM Liouville shoreline evolution model

Substitute one-line model to random differential equation, then a random process model of shoreline position can be given

$$\frac{\mathrm{d}y(x,t)}{\mathrm{d}t} = -H_{\mathrm{w}}\zeta(y,x,t) \quad (18)$$

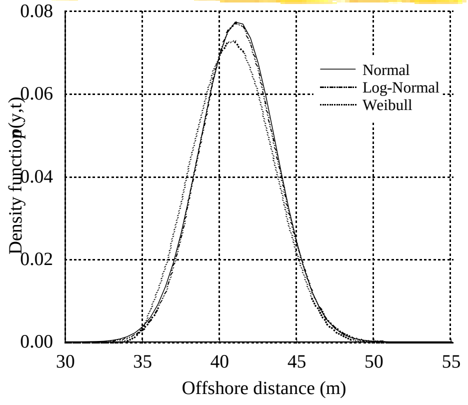
$$\zeta(y,t) = \frac{1}{16}\rho_{\mathrm{w}} g^{3/2} a_1 (\sin 2\alpha_{\mathrm{b}}^{\,i-1} - \sin 2\alpha_{\mathrm{b}}^{\,i})/(\Delta x) \quad (19)$$

In order to employ Liouville model Which can be reformulated as

$$\begin{cases} \dfrac{\mathrm{d}Z(x,t)}{\mathrm{d}t} = L(y,x,H_{\mathrm{w}}) \\ Z(x,0) = Z_0 \end{cases} \quad Z(x,t) = \begin{bmatrix} y(x,t) \\ H_{\mathrm{w}} \end{bmatrix} \quad L = \begin{bmatrix} \eta \\ 0 \end{bmatrix} \quad Z_0 = \begin{bmatrix} y_0 \\ H_{\mathrm{w}} \end{bmatrix} \quad \eta(y,t) = H_{\mathrm{w}}\zeta(y,t) \quad (20)$$

Part D SDEM Liouville Shoreline Evolution model

The probability density flow does not have sources, means



$$\partial \eta(y,t)/\partial y = 0$$ (Shvidler and Karasaki, 2003)

Eq. (20) can be simplified by a one-dimensional advection

$$\frac{\partial p_{YH_w}(y,h_w,t)}{\partial t} + \dot{Y}(h_w,t)\frac{\partial p_{YH_w}(y,h_w,t)}{\partial y} = 0 \qquad (21)$$

$\dot{Y}(h_w,t) = \partial \eta(y,t)/\partial y$ is the 'velocity' of the response for a prescribed

which means shoreline travels by this velocity

$p_{YH_w}(y,h_w,t)$ Joint PDF is numerically solvable

Flux limiter imposed by a total variation diminishing (TVD) (Harten, 1983; Sweby, 1985)

Publications

Liu Xia, Xingzheng WU, Junxia Xin, Hangong Tian. *"Three-dimensional stress and displacement analysis of Yutiao concrete faced rockfill dam". International Association for Second International Symposium on Flood Defence. Science Press, New York Ltd. 2002, 1494-1500.*

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谢谢各位同仁

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Thanks

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Cooperation

x.wu@dundee.ac.uk